On microwave modes in general and Brewster modes in particular

The normalised wavelength v

The overall mode properties of any cylindrical† waveguiding applicator or cavity of the TE and TM type‡ can be characterised with regard to its impedance (and by that wave reflection properties) by a single parameter; the *normalised wavelength v*:

$$v \equiv f_c/f = \sin\theta^i \qquad (1)$$

where θ^i is the equivalent angle of incidence in the geometric optics interpretation of mode behaviour, f the operation frequency and f_c the "cut-off" frequency below which no propagation of the mode occurs in an infinitely long waveguide.

Only a discrete set of modes may propagate in a waveguide, since the E field parallel to the walls must be zero. One may interpret this requirement by only patterns of constructive interference being possible. This geometric optics interpretation of mode behaviour gives the following relationships of the modes in a rectangular waveguide:

$$v^2 = (\tfrac{1}{2}\lambda_0)^2\cdot[(m/a)^2 + n/b)^2] \qquad (2)$$

where (m, n) are the horizontal mode (integer) indices, and (a, b) the corresponding horizontal dimensions of the rectangular cavity/applicator.

It is to be noted that theoretically very similar but mathematically more complex results are obtained for other waveguide cross sections such as circular.

An important relationship is the vertical wavelength λ_g of the mode:

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-v^2}} \qquad (3)$$

The loaded cavity; general

In order for analytical calculations to be possible, the load extends to all four cavity walls, as shown in Figure 1. The load is characterised by its relative complex permittivity $\varepsilon = \varepsilon' - j\varepsilon''$, where $\varepsilon''/\varepsilon' = \tan\delta$. ε' is the (relative real) permittivity ("dielectric constant"), ε'' is the loss factor and δ the loss angle.

A simplification made here is to consider the load so thick and lossy that there is no retroreflected wave from its bottom surface. If the load thickness is comparable to or larger than its penetration depth d_p (about 7…20 mm for most unfrozen foods at 2450 MHz), no essential changes occur. However, conditions become different if the horizontal cross section of the load is smaller. Therefore, the analysis is limited to cases where the load covers at least half the cavity cross section. Numerical modelling can be used to quantify the differences with regard to cavity mode behaviour, and it is then generally found that the approximations used here are quite satisfactory, for reasonably flat and large loads.

When the first downwards-going wave from the ceiling feed area hits the load surface, a part of the power is reflected upwards (by the load reflection factor r^-) and the remainder goes into the load (where it is subsequently absorbed). The upwards-going wave is then retroreflected in the ceiling area, with a reflection factor r^+. Since the definition of r is for the electric field, it becomes negative for a normal mode both at the load (having a lower impedance than that of the cavity volume mode) and at the ceiling (which is supposed to be essentially metallic, i.e. have a low impedance). The successive reflections and retroreflections build up a vertically standing wave, with an amplitude A in its maxima which becomes $(1-r^-)/(1-r^-\cdot r^+)$ times (larger) than the cavity input amplitude (set to 1). It can be shown that a cavity matched to the input line requires $r^- = r^+$. As an example, for $r^- = -0{,}7$ and a matched cavity, the amplitude becomes 3,3 times higher than the input signal in the cavity. For a

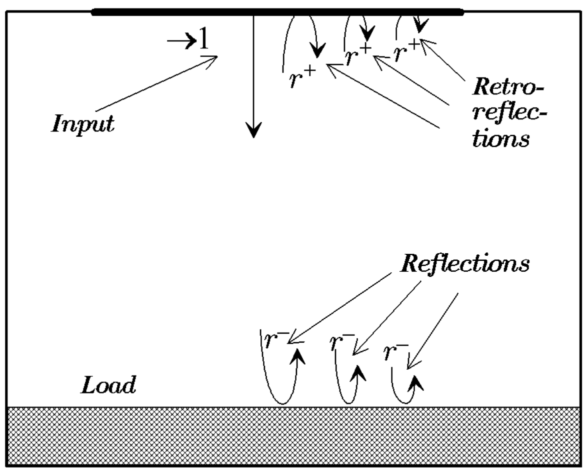

Figure 1. Reflection factors

† Cylindrical is used here mathematically: the horizontal cross section is constant along a z-directed axis. This means that for example circularly cylindrical applicators can also be used for the purposes dealt with here. The discussion is, however, limited here to rectangular applicators, for simplicity.

‡ TE modes have no E field component in the direction of propagation, and TM modes no such H component. Generally, this direction is z, and the major surface of the load is in a (constant) z plane.

weakly coupled cavity ($r^+ \approx -1$) the amplitude becomes 5,7 times higher. If there is instead destructive interference, this can be translated into $r^- = +0,7$ and the amplitude becomes only 0,18 times for the weakly coupled cavity. The resulting quotient of (5,7/0,18) ≈32 times is reduced some by the retroreflection in the ceiling, but the power density quotient becomes this factor squared. It is obvious that only resonant or low-r^- conditions will provide efficient power transfer from the cavity input area to the load in typical microwave oven cavities.

The need for resonance creates another important restriction of v (in addition to the "horizontal" restriction by equation (2)) and thus the number of possible cavity modes as compared to the possible vertically propagating waveguide modes in an open-ended waveguide.

In the mode terminology normally used, a third mode index p is used to characterise the standing wave field variations of a resonant mode along the vertical (z) direction. In an empty cavity, the p index for TM modes can then be integers 0,1, 2, …, where p is just the number of $\frac{1}{2}\lambda_g$ in the z direction. Since the ceiling and bottom of the cavity are plane metal surfaces they have approximately zero wave impedance.

With a typical dielectric load in a cavity with a typical TM mode, the load impedance is lower than η_g if v is reasonably low, and indices may be 1, 2, … . Since the load is supposed to be lossy and not have zero impedance as has the metal bottom, the zero index situation becomes rather undefined and may not be of practical interest.

Impedance relationships and pseudo-Brewster modes

The important wave impedance relationship for TM modes becomes

$$\eta_g = \eta_0 \cdot \sqrt{\varepsilon - v^2/\varepsilon} \qquad (4)$$

where η_0 is the wave impedance of free space and ε the relative permittivity of the region; $\varepsilon = 1$ in the cavity/applicator space.

By setting η_g equal in the load with permittivity ε to that of the cavity space (with $\varepsilon = 1$), the equation gives the condition for equal impedances and by that reflectionless transmission into the load, as

$$v_B^2 = \frac{\varepsilon}{\varepsilon + 1} \qquad (5)$$

where subscript B is for the so-called Brewster (reflectionless) condition.

Since v_B corresponds to $\sin\theta_B$ in the electromagnetic ray description of waveguide modes, it can easily be shown by some trigonometric calculations that the Brewster incidence angle is

$$\theta_B = \arctan\sqrt{\varepsilon} \qquad (6)$$

v has to be real for a cavity volume mode and perfectly reflectionless conditions can thus not be achieved for lossy loads. However, as is the case for the corresponding plane wave, minimum reflection factors (*pseudo-Brewster conditions*) can be obtained also for high-loss loads. $|\varepsilon|$ is then suitable to use and gives the proper v also for high quotients $\varepsilon''/\varepsilon'$. The minimum reflection factor can be deduced by using Snell's law and the equivalent Fresnel cosine formula for TM-polarised plane waves. One obtains the reflected power at the *pseudo-Brewster condition* θ^i_B

$$\mathcal{P}^r/\mathcal{P}^i = (\delta/4)^2 \cdot (1 - 2/|\varepsilon|) \qquad (\text{at } \theta^i_B) \qquad (7)$$

where δ is in radians, superscript i is for incident and r for reflected. It is seen that $\mathcal{P}^r/\mathcal{P}^i$ becomes quite small for typical workloads. The formula gives a practically insignificant error even for $\tan\delta$ up to 3, which is much higher than in any typical load considered here.

As examples, the following (frequency-independent) data are obtained for the Brewster conditions, from equations (3) and (5):

Brewster data for TM modes			
$\lambda_{B,g}/\lambda_0$	$\lvert\varepsilon_B\rvert$	θ^i_B (°)	v_B
6	35	80½	0,986
5	24	78½	0,980
4	15	75½	0,968
3	8	70½	0,943
2	3	60	0,866

It is seen from the table (and can readily be shown by equations (3) and (5)) that the relationship between $\lambda_{B,g}/\lambda_0$ and $|\varepsilon_B|$ becomes quite simple:

$$\lambda_{B,g}/\lambda_0 = \sqrt{|\varepsilon_B| + 1} \qquad (8)$$

$\lambda_{B,g}/\lambda_0$ becomes quite high for ε values typical of many load substances with a high water content. The v interval of interest is rather narrow and close to the "cut-off" limit $v = 1$.

Brewster modes are not resonant since they do not have any vertically standing wave in the cavity/applicator. No p value can thus be assigned. Instead, a last subscript B may be used.