Some aspects on the microwave penetration depths and dielectric properties of foods.

Basic concepts and definitions

The microwave properties of foods and many other materials which are heated in microwave ovens and equipment are described by what is called the dielectric constant and dielectric loss factor. These quantities were used very early in electrical and electronic engineering and are quite suitable for characterisation of for example electric capacitor filling materials. In optics, the index of refraction is instead used, and the power losses in the material are described by the so-called extinction coefficient. If there are no losses, the dielectric constant is simply the square of the index of refraction. This was actually Maxwell's most successful prediction after he completed the set of equations which now bear his name.

If losses are included, the combination of dielectric constant (ε') and loss factor (ε'') is called the relative permittivity and expressed as a complex number $\varepsilon = \varepsilon' - j\varepsilon''$, where $j = \sqrt{-1}$. The real part of the refractive index is labelled n and the imaginary part (the extinction coefficient) is often labelled $\varkappa$. The relationship between them becomes $\varepsilon - j\varepsilon'' = (n - j\varkappa)^2$.

> **Penetration depth**
>
> The depth below the surface at which 1/e (≈ 37 %) of the surface power density of a perpendicularly incident wave remains in an halfspace of the material, at a specified frequency.

The power penetration depth is almost always labelled d_p. The exact formula for calculating d_p becomes very simple if one uses the refractive index units: $d_p = \lambda_0/(4\pi\varkappa)$, where λ_0 is the free space wavelength of the microwaves. If instead the dielectric units are used, a very complicated formula results. That formula is in error in an astonishing number of textbooks but there is fortunately no need to reproduce it here, for two reasons: many handheld calculators can directly take roots of complex numbers—so that the simple formula $d_p = -\lambda_0/(4\pi \mathrm{Im}\sqrt{\varepsilon})$ can be used—and the error by using the approximate formula $d_p = \lambda_0\sqrt{\varepsilon'}/(2\pi\varepsilon'')$ is less than 3 % if the quotient $\varepsilon''/\varepsilon'$ is less than 0.5. The latter condition is fulfilled by a vast majority of food substances.

The conditions under which d_p fully applies are very idealised:

- There is a plane wave (the source has to be "far" away)
- This plane wave impinges perpendicularly on the dielectric object
- The dielectric object has a plane and very large surface, and it is very thick

The remainder of this article is about how well these conditions actually apply for heating in a microwave oven, and how more realistic conditions influence what happens. We will begin with the direction of incidence.

Microwave power flow and polarisation in relation to the load

Figure 1 shows a rather idealised situation: a plane microwave influencing a rather large solid dielectric cylinder. It is infinitely long in the direction perpendicular to the paper.

There are two distinct *polarisations* of the microwaves: TE (the electric field is in the plane of the paper) and TM (the electric field is perpendicular to the paper). Note that the reference direction is the cylinder axis here. That these two waves behave differently is clearly seen in Figure 2, which shows the result of a nowadays quite simple computer calculation using the exact mathematical solution to this old and well known problem of *diffraction by a dielectric cylinder*. The diameter of the cylinder is 200 mm and its dielectric data are ε=64–j16. This corresponds to for example cod muscle. The frequency is 2450 MHz. The figure shows the power intensity at the surface, from 0° to 180° positions; the pattern is of course symmetrical over the "top" part back to 0°. Since d_p is only 10 mm, there are no waves going across the cylinder inside it.

The TM-polarised wave gives only a hot area facing the direction from where the wave comes. The TE-polarised wave has three distinctive features, which will are actually of considerable importance also for the way "good" microwave ovens work:

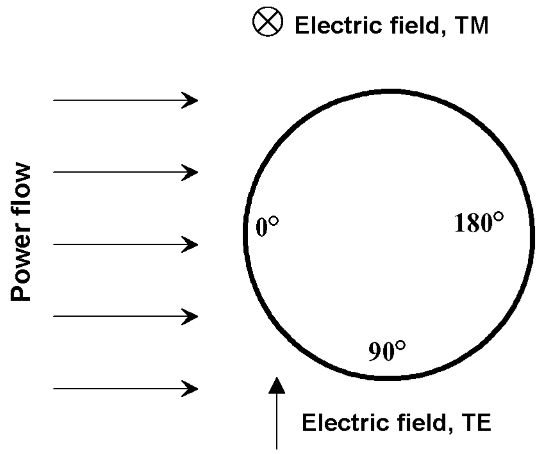

Figure 1

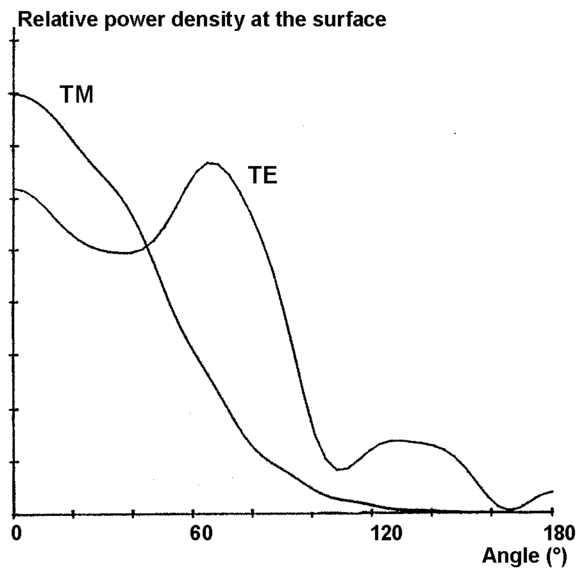

Figure 2. Power density at the surface of a cylinder of 200 mm diameter, with a permittivity of 64–j16

- There is a strong maximum at 70°, in spite of the quite oblique angle of incidence. It is due to the so-called *Brewster phenomenon*: at a certain angle of incidence, which is with a good accuracy determined by ε', the wave absorption becomes almost complete, even if the load has a quite high ε''. In the case of a plane wave towards a plane dielectric surface, the Brewster angle between the electric field direction and the surface plane is calculated by the arctangent of $\sqrt{\varepsilon}$ (for ε'=64 this becomes 83°; it has to be somewhat smaller in the case here due to the intensity reduction per surface area caused by the striking incidence).
- The maximum at the facing part is weaker than that at 70°, in spite of the straight incidence. This effect must be due to the curvature direction in relation to that of the electric field. It turns out that the heating of a curved part with smaller radius becomes even stronger for TM polarisation, and even weaker for TE polarisation. When the curvature is so strong that there is a sharp edge of the load, the electric TM-polarised field is thus parallel to the edge and the electric TE-polarised field is perpendicular to it. The heating at this edge may theoretically become more than 20 times stronger with the TM-polarised wave! In practical situations this *edge overheating effect* is indeed one of the most severe problems of uneven heating. Since there is no edge overheating for TE-polarised waves, the common explanation that waves from different directions add up at edges and causes the overheating is wrong. Instead, pure diffraction phenomena determine what happens, and this is very different for TM- and TE-polarised waves.
- The wave obviously follows the curved surface, since the heating is significant also in the shadow zone beyond 90°. This wave trapping by the load surface is indeed also of practical importance in microwave ovens, since it makes it possible to achieve an *under-heating effect* of the underside of reasonably large flat and thick loads. The wave is guided in the region between

the load and cavity bottom so that a large portion of the load becomes heated, rather than just a small region at the food edge as will a wave of TM polarisation. This ability is one of the crucial properties of "good" ovens.

It is indeed a remarkable feature of microwave oven technology that all these three advantageous properties of the system are promoted by the same wave polarizations: there should be no strong horizontal electric field components in the oven, but instead strong vertical components.

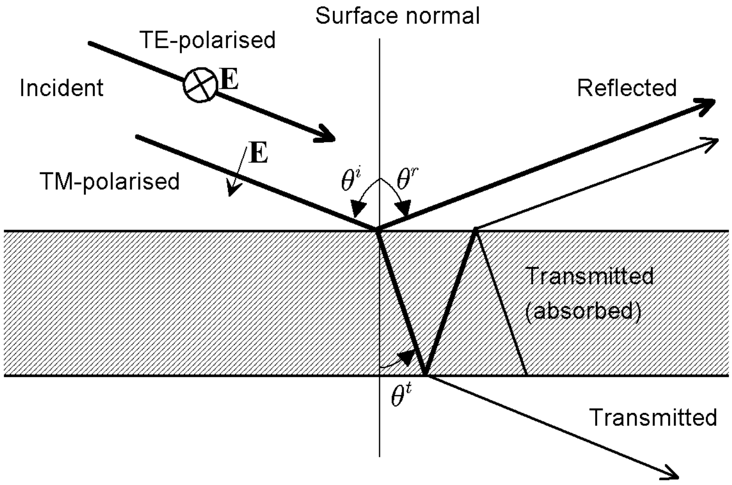

Figure 3. "Optical" plane wave polarizations and angles

Oven cavity modes and polarizations

For historical reasons, the polarisation notation is the same for cavity modes and for (optical) plane waves moving towards plane surfaces, but differs from that used above for cylinders and edges. The optics notation is shown in Figure 3. The desirable waves for good efficiency, reduced edge overheating and possibilities for under-heating are thus TM-polarized with reference to a vertical direction, with a large incidence angle θ^i, as illustrated.

Modes in rectangular cavities can be interpreted and analysed by what is called the electromagnetic ray concept, which uses sets of normally four plane waves in phase, with the same incidence angle θ^i as seen from the sides, and four symmetrical side angles as seen from above. Standing waves as seen from the sides are then built up. A very important relationship then obtained is that the vertical wavelength λ_g of the mode in the cavity airspace simply becomes $\lambda_0/\cos\theta^i$. If θ^i is large, λ_g also becomes large.

The reader may wonder if real microwave ovens can be analysed with respect to some kind of equivalent TM mode incidence angle, i.e. if there is actually some dominance of large θ^i in some ovens, and if there is some correlation to evenness of heating. There are indeed such methods, but no results have yet been published. The equivalent TM-polarized θ^i ranges from about 50° up to over 70° in typical microwave ovens, with a good correlation to the strength of edge overheating and under-heating.

Absorption depths

> ***Absorption depth***
>
> The depth below the surface at which 1/e (≈ 37 %) of the internal power density remains in the body of the material, at a specified frequency, load geometry and microwave mode.

It can be deduced from Figure 3 that the wave moves straight into the load only when the incidence is perpendicular. It is not difficult to show that the *absorption depth* d_a of the oblique modes becomes the same for TM and TE, and is obtained by replacing ε by $(\varepsilon - \sin^2\theta^i)$ in the standard formula for d_p. There is not much difference between d_p and d_a if ε is reasonably large. For $\varepsilon = 4 - j2$ and $52 - j20$ at 2450 MHz d_p is about 20,04 and 7,15 mm, respectively. At $\theta^i = 80°$ d_a becomes 17,77 and 6,90 mm, respectively.

There are some investigation reports in the literature on slabs such as that in Figure 3. Standing waves are created in the slab if its thickness is about or less than $3d_p$. However, only cases of slabs in free space have been studied and there has been no consideration of the practical necessity of having a metal wall below the load. It is of very significant practical importance that the vertical wavelength of modes with high θ^i becomes quite large in the cavity space, and that the Brewster phenomenon also works from the load and out: there is almost no reflection by the bottom surface of the slab of the wave going out of it but instead by the cavity bottom. This results in the creation of a standing wave pattern in the load to be mainly determined by the metal wall and not by any internal standing waves in the load itself. The typical consequence is that the distance to the cavity bottom becomes only such a small fraction of this vertical air wavelength away from the bottom of the load, so that the bottom part of this becomes virtually short-circuited. An illustration is given in Figure 4: the heating pattern in the load becomes very similar to that with a directly short-circuited slab. This leaves only about $\frac{1}{4}$ of the total power deposition in the lower half of the load. Were it not for the existence of the under-heating modes, unacceptable results would occur quite often in many microwave ovens, particularly with low-ε loads.

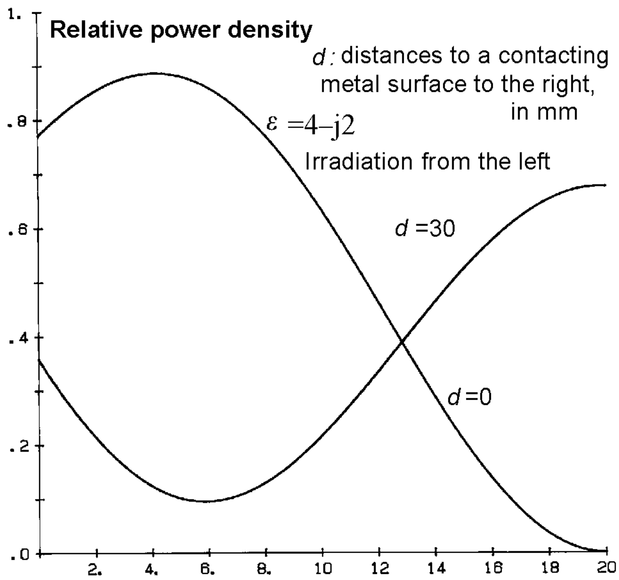

Figure 4. Power density pattern in a 20 mm thick slab under θ=0 incidence

Some quite intriguing phenomena occur with the under-heating modes, and may cause d_a to become *larger* than d_p. There are no simple formulas, but the phenomenon may be significant only for low-ε materials. As an example, for a thick slab of a material with ε=4–j2 placed 20 mm above the metal bottom of the cavity (no shelf), d_a (vertically upwards) becomes 9 % larger than d_p.

For cylinders in free space there must obviously be some small diameter for which internal standing waves dominate so that the absorption depth concept becomes useless. There may also be another effect caused by the curvature which increases d_a over also quite small d_p values. Both these phenomena can be quantitatively studied in Figures 5a and 5b, which are for the cylindrical TM and TE cases, respectively. Exact mathematical methods have been used to calculate the power density along the radius facing the plane wave incidence direction in a series of dielectric cylinders in free space, having different diameters but the same ε (64–j16, resulting in d_p=9,8 mm at 2450 MHz).

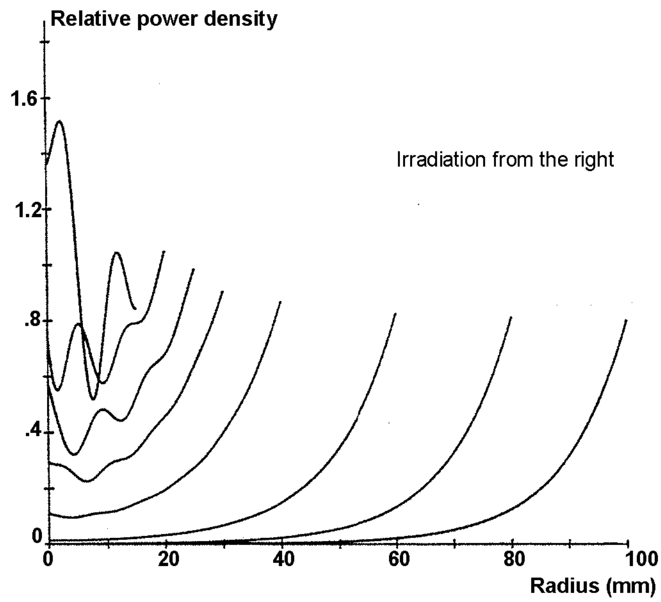

Figure 5a. Power density for TM incidence, in a dielectric cylinder with permittivity 64—j16, along the radius facing the incidence, with the radius as parameter.

Apart from the power density for small diameters being higher in the TM case as previously discussed, the internal standing wave phenomena become stronger and occur for larger diameters for TE. There are significant standing waves for 30 mm radius and smaller for TE, and for 25 mm and smaller for TM.

The curves in Figures 5a and 5b should of course become the same for very large diameters. However, the power density is less in the facing region also for 100 mm radius (note the differences in vertical scale in Figure 5a and 5b). The differences become even more significant already for about 60 mm radius and d_a then begins to increase for TE but not for TM, with decreasing radius. At 40 mm radius d_a has increased by 25 % for TE, but still not for TM, for which the radius has to go down to less than 30 mm for d_a to increase. These data apply to cylinders with $\varepsilon = 64–j16$ but very similar results are obtained for other food-like loads.

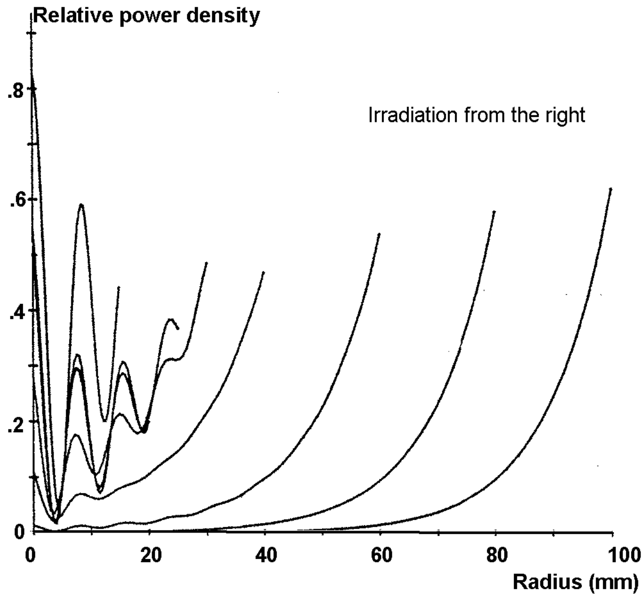

Figure 5b. Power density for TE incidence, in a dielectric cylinder with permittivity 64—j16, along the radius facing the incidence, with the radius as parameter.

Internal standing waves and their consequences

As seen in figures 4 and 5, the distance between consecutive internal standing waves becomes about $\frac{1}{2}\lambda_0/\sqrt{\varepsilon'}$, as expected from the theory. It can be said that this is the second of the two basic phenomena which can be quantified by ε' (the first one is the surface reflection and includes the Brewster phenomenon). Since the actual distances between power maxima and power minima become quite short for high ε', the times needed for temperature equilibration in relation to the heating times is of interest.

The first example is taken from Figure 5b. The distance between internal cold and hot regions become about 4 mm. The material is supposed to be compact and with a high water content, and can therefore be compared with practical data for boiling of eggs: it takes about 5 minutes for the centre to reach about $\frac{2}{3}$ of the surface temperature rise. Using the fact that the speed of equalisation is inversely proportional to the distance squared and that the egg radius is about 20 mm, the corresponding time for 4 mm distance becomes $5/5^2$ minutes, or 12 seconds. Using another rule of thumb, a heating time of five times the equilibration time does no harm. Hence, provided the food item is so large (and/or the microwave power so low) that the total heating time is at least one minute, the internal standing microwaves in compact food items with high water content are of no consequence.

The second example is taken from Figure 4. The dielectric properties may represent a frozen compact item, butter or some other solid fat, or a non-compact item such as bread. The distance between internal over- and underheated regions now becomes about 15 mm. If the heat conduction is comparable to that of non-frozen compact items, the minimum heating time becomes $5/(20/15)^2 \approx 3$ minutes. The problem of internal standing waves is a contributing reason for the need of recommendations on lower power settings for low-ε loads.

Finally, there may also be advantages of the internal standing waves created in rounded food items such as bake potatoes. Theoretical investigations have shown that if the size and dielectric properties and oven or applicator design are all favourable, a reasonably even heating throughout can be achieved in a cylindrical or elongated object having a diameter up to about 7 times d_p of the material! Interestingly, this rule is essentially independent of both ε' and d_p of the material, as well as the frequency.

Simultaneous heating of two loads with different dielectric properties

There have been misunderstandings about the influences on microwave heating by for example added salt to a food material. There have also been some incorrect statements on the relative heating rates of loads with dissimilar ε′ when heated simultaneously in microwave ovens.

The second issue is dealt with first and is illustrated in Figure 6. It can be shown that the overall incident power flow density (watts per surface unit) from the cavity space is the same towards a number of similar loads with similar geometry but dissimilar dielectric properties. Since no internal standing waves in the loads are considered, a further restriction is that all loads are thicker than about $3d_p$. Every working mode in the cavity has its own equivalent θ^i and power flux density, so all curves will in reality not be of equal basic amplitude as in the figure. However, the different loads will get a relative power by each mode in correspondence to their dielectric constant ε′.

It is concluded that the behaviour of an oven depends very much on the cavity mode composition and their relative strengths, but that much stronger standing waves, and by that a higher sensitivity to the precise dimensions, is obtained for high-ε loads with equivalent perpendicular incidence or TE polarisation. Statistically, this leads to less power being transmitted into the load by such modes in a multimode cavity. With high-θ^i TM modes, a quite ε-independent absorption results. It is then to be noted that the power density is in watts per volume unit and not per weight unit. The actual temperature rise rate thus depends on the density and specific heat capacity of the material.

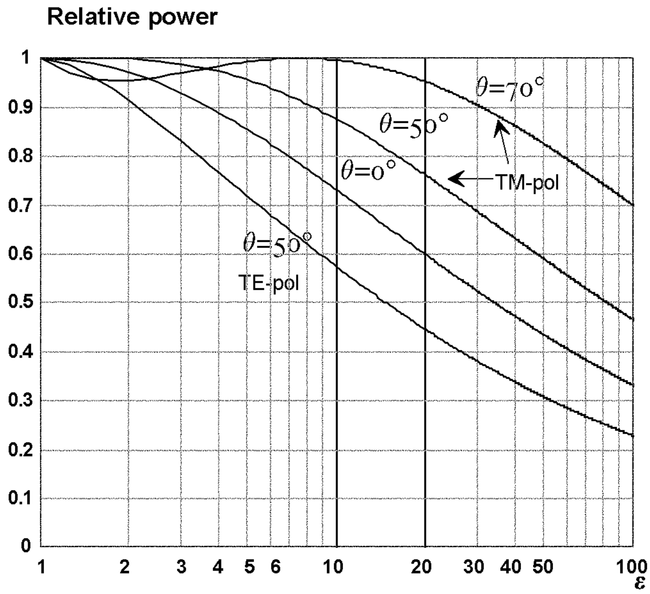

Figure 6. Power transmission into a dielectric, with the incidence angle as parameter

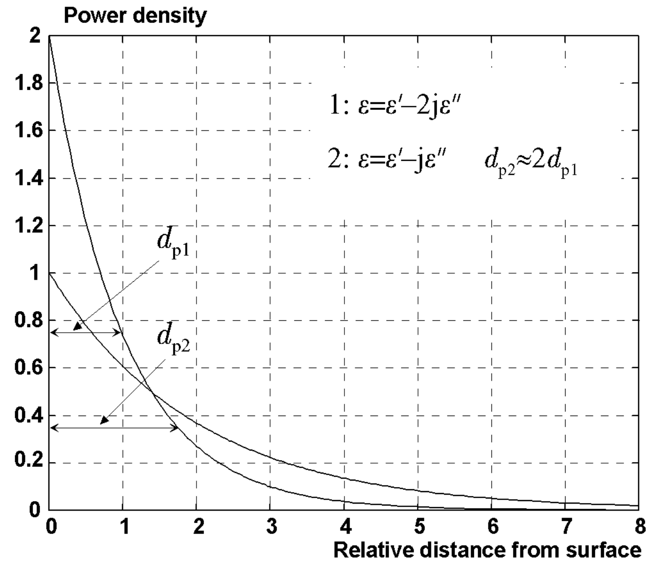

Figure 7. Power densities in loads with dissimilar loss factors

If two loads with about the same ε′ but different ε″ are heated together there will be no significant difference in overall absorbed power. However, d_p will be different. What happens is illustrated in Figure 7. The two materials have the same ε′, which results in the almost the same power reflection at the surface. As a consequence, the power absorption per surface unit of the two foods becomes almost the same. In the figure, this means that the surfaces under both curves is equal.

The power density is proportional to ε″ if the electric field strength is the same. This is the case only at the surface, resulting in twice the power density there, in the material with the doubled ε″. However, the stronger absorption results in a quicker “consumption” of the fields, so that the power density falls off with a d_p half as large as for the material with lower ε″.

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