A description of evanescent modes in open-ended applicators and cavities

The optical analogies of propagating and evanescent modes is first dealt with. Evanescent waveguide and open-ended applicator ("HERA") modes are then addressed. The text is intended for persons with some experience in mathematical physics, and provides some basics of the theory.

1 Propagating modes in a waveguide

Plane wave propagation is a very important case – but also very special. According to the basic theory of electromagnetism, all magnetic field lines are to be closed, but this happens only "in infinity" for plane waves. For such waves to be a realistic approximation, the distance from the source to the region under study, as well as the size of the region itself, must be very much larger than the wavelength. In all cases where the region under consideration is not infinitely larger than the source and the wavelength, the magnetic field lines form closed loops. The region where the electromagnetic fields exist is then limited and defined.

It is found that such fields under stationary time-harmonic conditions assume characteristic and discrete single or multiple (added) patterns. These can be of many kinds, examples being the single pattern in a coaxial line and the multiple interference pattern in large metal (multimode) cavities. Theoretically, there is only one solution to the wave equation in each such case, but this solution can almost always be separated into dependent, partially dependent or independent simpler *modes*.

Modes are each of the possible configurations of the fields in a given domain in space†.

In microwave heating applications, separation of the overall field pattern into modes is a very important tool for both understanding of mechanisms and for system synthesis.

The rectangular waveguide offers an application of an instructive method to develop mode characteristics and conditions for propagation. The waveguide walls can be considered to be perfect conductors. It then follows that there can be no parallel electric field component at the walls. This is used in the geometric approach below. The use of image sets of plane waves to represent waveguide modes is generally called the *electromagnetic ray concept*. The concept should not be considered to be "true" in a physical sense – it is merely a manipulation of the simple plane wave concept. That there is a deeper truth than that provided by the electromagnetic ray concept is shown by its failure to address the excitation problem. However, it provides an important link between plane waves and waveguide modes.

The geometry of a plane wave incidence situation,

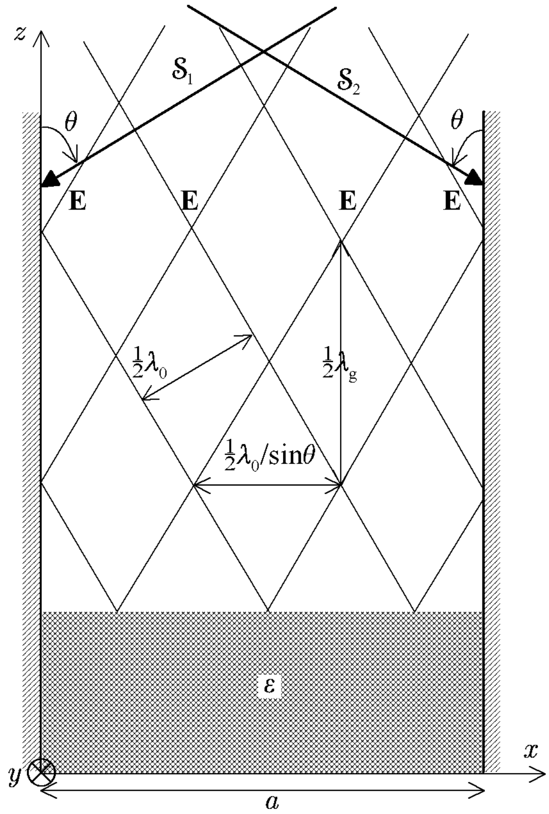

Figure 1. A metal trough with two TM-polarised rays

with two symmetrical incident rays S is shown in Figure 1. It shows a cross section (constant y plane) of a rectangular trough with width a in the x direction. It is considered infinite in the y direction ($b=\infty$) and is filled with a dielectric at the bottom, but any effects by the dielectric are not yet studied. The rays have polarisations (electric field – **E** – directions) perpendicular to the Poynting vectors S and are of the TM-polarised type‡ with the plane of the dielectric as

† This is the standard definition in the IEC Electrotechnical Vocabulary (IEV).

‡ Obliquely incident plane waves in free space can be separated into TE-polarised and TM-polarised waves. The latter are characterised by there being no magnetic (H) field in the (z) direction of propagation. **Only TM-type fields are of interest here.**

reference surface. The rays have incidence angles θ. The instantaneous E field lines are shown $\frac{1}{2}\lambda_0$ apart and represent moving positive and negative amplitude maxima. Since the waves are of constant frequency (time-harmonic), the two waves are to be vectorially added to represent the real situation. When this is made, the total field lines become as illustrated in Figure 2. The illustration represents an instantaneous situation of a propagating guided mode, with no reflected waves from the dielectric boundary. It is

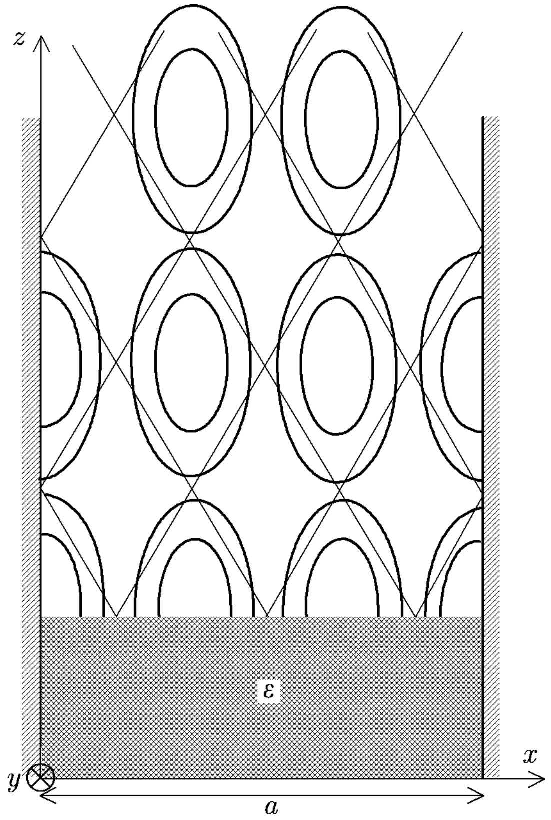

Figure 2. Vector E fields.

drawn in the same way as are waveguide mode illustrations in the microwave literature. This method may, however, lead to misunderstandings, since the *amplitudes* of the fields cannot be correctly described by closed lines with the amplitude being represented by their relative closeness. A more proper way of illustrating the fields is by quiver plots; see Figure 3.

The boundary condition at the metal walls require that there be no parallel ($E_{\|}$) component and thus restricts the relation between θ and a. Since the incidence angles θ are unchanged upon reflection, one obtains $a = m \cdot \frac{1}{2}\lambda_0/\sin\theta$, with $m = 3$ in the figure. m is an integer

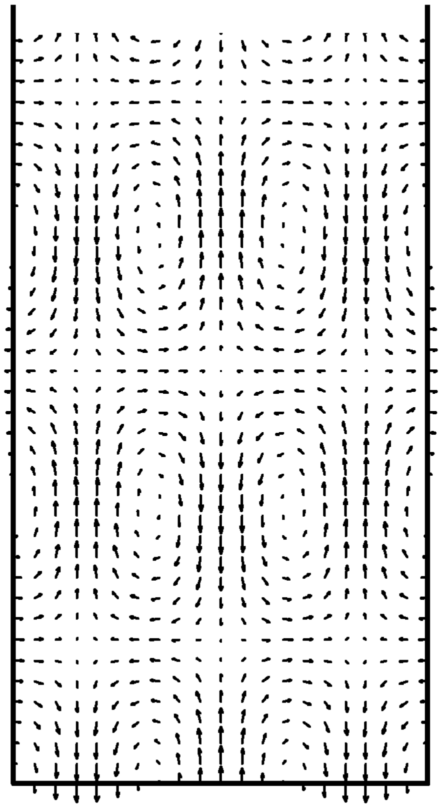

Figure 3. E fields in the waveguide of Figure 2.

and represents the number of x-directed standing halfwaves.

Of course, the waveguide may have an analogous field pattern also in the other vertical (constant x) plane. By relatively straightforward geometric calculations for the general case with four incident plane waves all having the same incidence angle θ and four symmetrical side angles φ, one obtains

$$\sin^2\theta = (\tfrac{1}{2}\lambda_0)^2 \cdot [(m/a)^2 + (n/b)^2] \qquad (1\text{-}1)$$

where n is the number of halfwaves and b the waveguide width, both in the y direction.

From figure 1, the z-directed wavelength of propagation of the waveguide mode becomes

$$\lambda_g = \lambda_0/\cos\theta \qquad (1\text{-}2)$$

For $\theta = 90°$ ($\sin\theta = 1$) propagation ceases if the waveguide is superconducting and infinitely long in the z direction. $\theta = 0$ corresponds to free space TEM propagation. It is concluded that $\sin\theta$ corresponds to λ_0/λ_c, where λ_c is the "cut-off" wavelength – the longest λ_0 allowing *normal propagation* in such a very long guide. Since λ_0 is inversely proportional to f, $\sin\theta$ also corresponds to f_c/f, where f_c is the waveguide frequency below which no propagation of the specified mode can occur in the infinitely long superconducting waveguide. In this border case, the mode becomes *evanescent* and λ_g becomes infinite.

The parameter $\sin\theta$ *is therefore of utmost importance in waveguide analysis, and is labelled the normalised wavelength* v†:

$$v \equiv f_c/f = \sin\theta \qquad (1\text{-}3)$$

Equations (1-1) and (1-2) then become

$$v^2 = (\tfrac{1}{2}\lambda_0)^2\cdot[(m/a)^2 + n/b)^2] \qquad (1\text{-}4)$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-v^2}} \qquad (1\text{-}5)$$

The normalised wavelength v is actually typically a short interval, since the operating frequency in heating applications is in a limited interval of – say – 2440 to 2460 MHz. Equation (5) has a limited number of integer solution pairs $(m;n)$ in each given interval of v. As a consequence, all possible combinations of $(m;n)$ – modes – for given values of a and b are represented by a finite set of v values.

There are of course also less intuitive and more formal methods to analyse waveguide modes than by the electromagnetic ray concept. Solutions to the electromagnetic wave equations in terms of analytical functions exist in several co-ordinate systems, such as the rectangular and circularly cylindrical, and are then obtained by the so-called separation of variables method. Three separation coefficients are obtained, each being dependent on only one of the co-ordinates. In the rectangular system these are x, y and z. If these separation coefficients are labelled k_x, k_y and k_z, the separation, eigenvalue or *wavenumber equation* becomes

$$k_x^{\,2} + k_y^{\,2} + k_z^{\,2} = k^2 \qquad (1\text{-}6)$$

where k is $k_0\sqrt{\varepsilon}$ in a region with relative permittivity ε and k_0 is the free space angular wavenumber $\omega/c_0 = 2\pi/\lambda_0$. The solutions are then elementary wave functions and are called *eigenfunctions*, which are expanded to give the equations for all field components. The k_x, k_y and k_z values are determined by the boundary conditions and called eigenvalues or just wavenumbers.

For the guides in the figures above, the eigenfunctions are all sine or cosine functions. With the waveguide cavity dimensions $x=0$ to a and $y=0$ to b equation (1-6) becomes

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = \left(\frac{2\pi}{\lambda_0}\right)^2 \qquad (1\text{-}7)$$

where m and n are integers as before, and λ_0 is the free space wavelength.

Returning to Figure 1 and the boundary between the trough space and the dielectric, it is found that k_x and k_y must each be the same in both regions, to fulfil the conditions of continuous H and tangential E fields everywhere at the horizontal boundary. Their sum $k_x^{\,2}+k_y^{\,2}$ *(and thus the normalised wavelength* v*, according to equation (4)) then has to be the same in all layers.* – This is, however, only valid exactly if two conditions are fulfilled: the dielectric has to fill the whole cross section; and there should be no direct (nearfield) influence by the source.

The propagation in the forward direction in a waveguide filled with a dielectric with relative (complex) permittivity ε is determined by an exponential function $\exp(-jk_z z)$; in the backward direction the function becomes $\exp(+jk_z z)$. The propagation equations (analogue to the equations for free space propagation) thus become

$$E(z) = E(0)\cdot\exp(j\omega t \mp jk_z\cdot z)$$

$$k_z = k_0\cdot\sqrt{\varepsilon - v^2} \qquad (1\text{-}8)$$

where the relative permittivity is 1 in the empty waveguide space and ε in its loaded part.

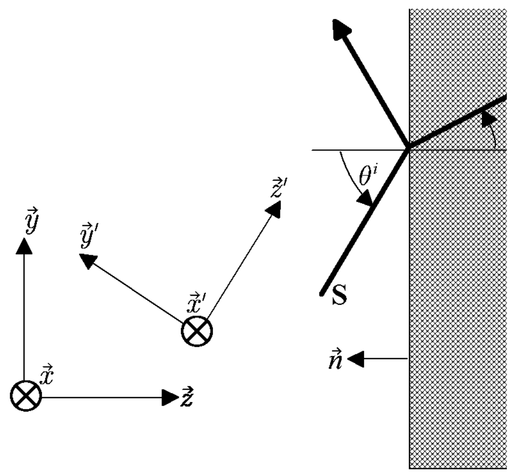

Figure 4. Coordinates (optical case)

† The notation v is specified in the IEC-27 standard. λ_r is given as an alternative, but the central importance of v justifies a simple, non-indexed notation, which also reduces the risk of confusion.

† A good description of eigenfunctions is given in *Harrington*.

2 *The "total reflection" phenomenon*

If the incident wave, with a large angle of incidence θ^i, comes from a region with higher ε, the law of refraction gives $\sin\theta^i > 1$. This means that $\cos\theta^i$ becomes imaginary. The negative root must be taken due to the convention: $\exp(-j\,k_0 \cos\theta^i z)$, since the field amplitude decays with z in the $+z$ direction in a physical system.

Waves with these properties exist and are called *evanescent*, as they do normally not propagate in the low-ε region. Formally, the generalised trigonometric functions are valid for what is often called complex angles. Of course, complex angles do not exist in the real world. A better interpretation is given by the normalised wavelength concept for modes in a metal waveguide (such as the microwave oven cavity) or along a reactive boundary.

> *The factor v which is $\sin\theta^i$ in the optical case actually characterises propagation in a more general way than just by a plane wave angle and can be any positive number. The wave becomes evanescent when v passes 1.*

Mathematically, $\arcsin v$ where v is real and > 1 is obtained as (arguments in radians; underlining signifies a complex quantity)

$$\sin\underline{\theta} = \sin(\mathrm{Re}\,\theta + j\cdot\mathrm{Im}\,\theta) = v \;;$$

$$\mathrm{Re}\,\theta = \tfrac{\pi}{2};\;\; \mathrm{Im}\,\theta = \ln(v - \sqrt{v^2-1}) \qquad v > 1 \qquad (2\text{-}1)$$

The cosine function is obtained from v, for the transmitted wave, as

$$\cos\theta^t = -j\sqrt{v^2-1} \qquad v > 1 \qquad (2\text{-}2)$$

The minus sign on the root is for decaying amplitude in the y and z directions (see Figure 4).

Equation (2-1) indicates that the evanescent field propagates along the surface in the y direction, with a decaying amplitude in the z direction. Formally, all equations for normal propagation can be used to calculate fields, impedances and boundary conditions, except in the singularity case $\cos\theta^t = 0$. That case is only mathematical, however (the function is continuous at $\theta^t = 90°$).

The simplest way to quantify the situation is by using equation (1-8). Inserting $v = \sin\theta^t$ and studying *the evanescent wave in the airspace* above the dielectric surface one then obtains the *decay distance* d_d – for both TE and TM polarisation:

$$d_d = \frac{-\lambda_0}{4\pi\cdot\mathrm{Im}\sqrt{1-v^2}} = \frac{\lambda_0}{4\pi\sqrt{v^2-1}} \qquad (2\text{-}3)$$

This is the distance in an empty and constant cross section waveguide over which the evanescent mode field amplitudes decay by a factor of $\sqrt{e}$ (to 60 %) and the energy density of the field by e (to 37 %).

As an example, the "critical" incidence angle inside the dielectric becomes 30° for $\varepsilon = 4$. Using $\theta^i = 45°$ one obtains $v = \sqrt{4}\cdot\sin 45° = \sqrt{2}$ and $d_d = 9{,}7$ mm at 2450 MHz. If instead $\varepsilon = 52 - j20$ the critical angle becomes only 7,7° and v at $\theta^i = 45°$ becomes 5,28. d_d becomes only 1,9 mm at 2450 MHz. It may be noted that the complex ε for a lossy substance should be used in the calculation of v and d_d and thus provides correct results. Evanescence in a region with $\varepsilon = 1$ always results in $\mathrm{Re}\,\theta^t = 90°$. With losses in it, $\mathrm{Re}\,\theta^t$ becomes smaller, indicating some power transmission out from the high-v dielectric – again only for very large dimensions compared to the wavelength. For the microwave situations considered here, the common terms "total reflection" and "cut-off" are thus somewhat unfortunate, since the fields may extend a practically significant distance away from the dielectric object from where the evanescent wave emanates. If there is a short low-ε region between two dielectric objects, power may "leak through", between the regions.

Finally, the small critical angle and short decay distance of evanescent fields outside high-ε objects show that confinement of energy to the objects can be almost complete if they are not close together.

> *This indicates that internal standing wave phenomena may occur more readily in thin high-ε dielectric objects which are wedge-shaped or have other non-parallel surfaces, than what may be concluded from simple theoretical results from slabs with parallel major surfaces.*

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3 *Zero index and evanescent modes*

For $v = 1$, some field components and the Poynting vector S vanish. This is, however, only a mathematical singularity and valid only for infinitely long superconducting waveguides.

> *A TM mode type having $v = 1$ is called zero index mode. When $v > 1$ the mode is evanescent.*

Means for excitation are necessary in practical systems. This is briefly dealt with later.

The normalised wavelength v becomes larger than 1 if a, b or the frequency is further reduced. The terms "cut-off" or "critical" are often used in transmission line engineering instead of evanescent, but is must be noted that the propagation disappears "suddenly" when the frequency is reduced only if the waveguide is superconducting and infinitely long. If there is a load which can absorb power, it will do that if the distance to the source through the waveguide is of the same order as that over which the mode field intensity decays by a factor e. As an example, if $\nu = 1{,}01$ for a waveguide mode at 2,45 GHz, the real part of $jk_z \cdot z$ becomes 1 for 137 mm; the field intensity has thus decayed by e over that distance. – It can be shown that evanescent modes with about this ν and even somewhat larger value may transfer a significant part of the power over distances more than 200 mm in microwave cavities. The interesting properties and potential uses of evanescent resonant TM modes for microwave heating applications are underestimated. The terms "cut-off" and "critical" are thus indeed misleading for studies of heating applicators and cavities.

Evanescent resonant modes

A basic condition for resonance is that the inductive and capacitive field energies in the system are equal. But an evanescent TM mode has an excess capacitive (i.e. electric field) energy, which is manifested by its strong component in the direction of evanescence.

A simple mechanism – and actually that in the first literature report on evanescent resonance in an unexplained illustration of a computational finding[†] – can occur with lossy dielectric loads. This is because the Poynting vector, which becomes $\sqrt{\varepsilon - \nu^2}/\varepsilon$, has a positive imaginary component in such loads. They are thus inductive. But these phenomena are quite weak. They can, however, be amplified somewhat if the load is thin (as it was in the literature report).

In the proprietary applicator (HERA[‡]) systems, inductive means in the applicator ceiling feed region are instead used. The mode d_d is about the same as the applicator height – slightly less than λ_0.

Properties of the modes in the HERA systems

These systems offer a series of major advantages:

- an unusually good and permittivity-independent system impedance matching (high efficiency for a variable load, during for example drying)
- a very good thickness-independent system impedance matching (high efficiency)
- the strongest possible heating by the perpendicular $E_\perp$ component, resulting in a more even horizontal heating pattern of drying or semidry loads
- very efficient use of the metal bottom of the cavity or tunnel, to transform microwave energy "falling between load items" to create strong so-called under-heating (LSM) modes[⹋] that improve the overall heating evenness

The top two items have been found to function remarkably well. They are explained as follows:

1. Relatively much oscillating energy is confined in the excitation region at the applicator ceiling, and thus not evenly distributed everywhere as in a multimode system.
2. The feed system acts as a field filter, effectively prohibiting waves reflected back to the ceiling region to enter the magnetron waveguide unless their pattern corresponds to those of the waves sent out into the applicator. Other wave energy is just reflected back down towards the load.
3. The pattern characteristics of a returning wave that can enter the magnetron waveguide must be that of the primary evanescent mode. But such a returning wave becomes evanescent upwards into the applicator, and therefore becomes very weakened on its way. – Together with item 1. above, the result is that a very small fraction of the energy that has passed out from the applicator can find its way back into the magnetron waveguide.

The factors above also explain the extremely low power coupling between closely adjacent applicators – typically 0,5 % or less. This makes it possible to assemble multiple applicators with common walls, into large tunnel ovens. But the single mode heating pattern characteristics typically necessitate staggering.

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† F Paoloni. 1989. "Calculation of Power Deposition in a Highly Overmoded Rectangular cavity with Dielectric Loss", Journal of Microwave Power and Electromagnetic Energy, (24):1, p.21-32.

‡ *H*ybrid *E*vanescent *R*esonant *A*pplicator. See e.g. patent application WO2005022956 (A1), available at e.g *http://ep.espacenet.com*

⹋ See: Risman, P O. 1994. "Confined modes between a lossy slab load and a metal plane as determined by a waveguide trough model". JMPEE 1994 Vol 29 No 3 p 161-170