Accurate microwave permittivity data of liquid water between –20 °C and +100 °C

Data between –15 °C and +40 °C were presented at the 6th ISEMA Conference in Weimar, May–June 2005. Measurements and retromodelling has now been completed up to +90 °C, and were published in an April 2007 special issue of Measurement Science and Technology. *The measurement applicator (brass) is shown on the homepage. Instrumentation was made available at SIK, Göteborg Sweden and assistance was provided by Ms Birgitta Raaholt there. Neither the applicator nor this project would exist were it not for the excellent Quickwave™ modelling software (www.qwed.eu).*

Differences among literature data are rather small from +10 °C up to about +60 °C, above which modern literature data is scarce. The present data are in very good agreement with critically selected literature data. Differences among literature data are comparatively larger at 0 °C and below, and there are very few reports.

The present measurements employed a complete multi-step numerical modelling of a dual resonant cavity at about 920 MHz and 2230 MHz. The measured data at about +20 °C are used as calibration for the modelling and calculations of data at other temperatures. Due to the high resolution and considerations of various error sources, the resulting accuracy becomes high and allows the construction of improved empirical formulæ for the Debye relaxation behaviour.

The Debye relaxation equation

This is $$\varepsilon = \varepsilon_\infty + \frac{\varepsilon_S - \varepsilon_\infty}{1 + j \cdot \left(\frac{f}{f_D}\right)} \quad (1)$$

where

f is the frequency;
ε is the complex (relative) permittivity $\varepsilon'-j\varepsilon''$;
ε_s is the static permittivity ($f\to0$);
ε_∞ is the (fictitious) permittivity for $f\to\infty$; and
f_D is the Debye relaxation frequency

A distribution parameter α may be introduced, but there is a consensus that this is not applicable to water. The present investigation also supports this.

With known values of ε_s, ε_∞ and f_D one may calculate the complex ε for any frequency f, using equation (1), on a calculator handling complex numbers.

Basic assumptions and inaccuracies

The measurement method relies on using a given set of ε_s, ε_∞ and f_D at one given temperature, to obtain the sample tube diameter and certain cavity conductivity variation corrections. Based on a detailed literature survey, the following basic data was used (note that ε_∞ is not sensitive if $f \ll f_D$):

$$\varepsilon_s = (80{,}30 \pm 0{,}10) \qquad \varepsilon_\infty = (5{,}0 \pm 0{,}2) \qquad \alpha = 0$$
$$f_D = (16{,}8 \pm 0{,}6)\ \text{GHz} \qquad t = 20{,}00\ °\text{C} \quad (2)$$

The error limits given above represent the true absolute data with a very high certainty. The nominal values are used in the following.

The measurements were made in two frequency bands: about 920 and 2230 MHz. The accuracy is therefore *best from very low frequencies up to about 3 GHz.* An additional uncertainly related to ε_∞ and f_D may result in up to 1 % further errors at 6…8 GHz.

The *temperature inaccuracy was less than ±0,2K.*

Error limits of the equations are given at the end of next section.

Debye relaxation data

$$\varepsilon_s(t) = 1/[(0{,}0112844 + t\cdot5{,}81145\text{E}{-5} + t^3\cdot9{,}8178\text{E}{-10})] \quad (t \text{ in } °C) \quad (3)$$

$$f_D(t) = \exp[(2{,}18787 + t\cdot0{,}05247)/(1 + t\cdot0{,}0073768)] \quad (t \text{ in } °C) \quad (4)$$

$$\varepsilon_\infty = 5{,}0 \quad (\text{no temperature dependence}) \quad (5)$$

The absolute error in ε_s for $t < +5$ °C is less than 0,7 units, including that in equation (2). For $t > +5$ °C the added error to that of the reference value at +20 °C is < 0,2 units. Note that a temperature difference of 1 K gives $\Delta\varepsilon_s \approx 0{,}3$.

The absolute error in f_D may be up to 4 % for $t < -10$ °C, but is less than 2,5 % for $-10 < t < +5$ °C. It is less than 1,5 % for $t > +5$ °C.

The resulting errors in ε' in the frequency range 0 to 3 GHz is the same as in ε_s. For ε'', the error may be up to 3 % for the worst cases of temperature and frequency.

These error limits are less than half of what has been reported in the literature up to now, for t> +5 °C. For t<–5 °C only one earlier literature reference exists and has a still larger relative error.

Calculated data at 915 MHz and 2,45 GHz

Temp. °C	915 MHz ε'	915 MHz ε''	2,45 GHz ε'	2,45 GHz ε''
–20	93,73	21,36	71,33	42,74
–15	92,91	16,65	77,44	36,74
–10	91,44	13,13	80,88	30,87
–5	89,66	10,49	82,44	25,69
0	87,75	8,49	82,75	21,36
10	83,85	5,77	81,34	14,96
eq.2:20	*80,07*	*4,09*	*78,72*	*10,76*
30	76,48	3,01	75,71	7,96
40	73,07	2,27	72,60	6,05
50	69,82	1,76	69,53	4,70
60	66,71	1,40	66,52	3,73
70	63,72	1,126	63,58	3,01
80	60,83	0,921	60,73	2,46
90	58,03	0,763	57,96	2,04
100	55,31	0,638	55,26	1,71